RADIOACTIVE TIMEKEEPING

This invention comprises methodology and apparatus for determining consecutive invariant UNIVERSAL ABSOLUTE TIME-INTERVALS, where ABSOLUTE is defined as invariant everywhere at all times and UNIVERSAL is defined as throughout the whole Universe at all velocities and accelerations. The invention uses cumulative counting of the number of individual emissions from the radioactive source, and a means of computation with natural logarithms to measure exponential decay. The invention avoids the use of non-ABSOLUTE time-intervals. It does not require the measurement of the number of emissions per second. The invention allows DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETERS OF DECAY of radioactive species to be determined. The invention requires the radioactive source to remain above the STOCHASTIC THRESHOLD MASS throughout the operating lifetime of the apparatus.

Timekeeping relies on phenomena which cause the regular repetition of events in series long enough to run in parallel with the events which are being timed. First there were water clocks, which relied on gravity and fluid flow, and sundials, which relied on the daily spinning of the Earth on its axis and its orbit around the sun. Mechanical clocks which followed had the problems of the friction of bearings and of supplying a constant input of motive power. ‘Frictionless bearings’ and the pendulum and escapement mechanisms alleviated these to a large extent, but the clocks could not measure time-intervals with much greater precision than the second.

The measurement of time-intervals has now assumed great importance, because modern processes require much greater precision. The newest sources of regularly repeated events are the vibrations of crystals, and, most recently, the transitions in atomic species which can be measured with great precision. Thus the SI unit of time-interval is the second, which is now defined as the duration of 9 192 632 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom. With an atomic clock 1 part in 10^9 can be measured, and timekeeping to within a few seconds per century has been claimed.

Nevertheless, all such clocks, whatever their precision, give units of time-interval which are characteristic of the physical phenomena on which they are based, and which may vary independently and systematically with the environment. Thus mechanical, astronomical and electromagnetic ‘seconds’ are all different in length, even though they were intended to be identical when they were first agreed. This causes problems particularly when aspects of different technologies come together in common projects.

Variations in the wavelength of electromagnetic radiation are a potential source of problems for the SI standard of time-interval, since it is defined in terms of periods of radiation. Computers use an average of the times kept by several atomic clocks in laboratories scattered around the world to smooth out the differences, although they each separately express their times with great precision.

Systematic variations occur in space. Time-intervals on GPS satellites are different from time-intervals on Earth i.e. the clocks run at a different rate, and this has to be taken into account when computing positions on the ground. In addition, significant variations in timekeeping were observed when caesium-133 clocks were flown around the equator in opposite directions. When flown in one direction they gained time, when flown in the other they lost time.

Increases of wavelength have variously been ascribed to the recession of the emitters (stars), time dilation and gravity (the Einstein redshift). There is even the suggestion that time-intervals may lengthen with velocity, as in Relativity. Back on Earth reductions of wavelength certainly occur when light travels through transparent media such as liquids or glass, because frequency is maintained while velocity is reduced.

All the different methods of measuring time-intervals will continue gradually to move out of synchronisation, because the phenomena on which they are based are varying differently over time, often as long period oscillations. As a result periodic realignments will be needed, rather like the historical adjustments to the calendar.

There is the further potential problem for the international system of measurements that the SI unit of length, the metre, has also been defined in terms of electromagnetic radiation, and hence time, since it involves the velocity of light, the distance travelled per second. The metre is defined as equal to 1 650 763.73 wavelengths of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom. Any phenomenon which affects time-interval must also affect the SI unit of length, the metre.

The danger of using wavelengths of electromagnetic radiation to define both the second and the metre was raised in the mid 1950s when they were first proposed as standards. The objection was that units of length and of time would lose their independence, and frequent comparison was urged between the new units and the physical metre and astronomical second. However, this would not overcome the problem of periodic realignment with the turmoil and economic costs this involves.

All this casts some doubt on whether electromagnetic phenomena should be regarded as ABSOLUTE measures of time-intervals in spite of their precision.

Previous improvements in timekeeping have been directed towards more precise and reliable methods of measuring the second. Originally the length of a second was defined with respect to the day. The day was, and still is, defined as 24 hours of equal length, each of which consisted of 60 minutes of equal length, and each minute in its turn consisted of 60 seconds of equal length. The number of seconds of equal length in a day was thus precisely calculable. This specified the number of seconds in a day exactly.

However, the length of a day measured by the highest point of the sun in fact oscillates over the period of a few weeks because of the variations of the movement of the Earth in space with respect to the Sun. Since the day was by definition always an exact number of seconds, the length of a second varied from day to day. If the second was kept at a fixed length, as on a clock, the time at which midday occurred, measured by the height of the sun, changed from day to day. This had to be allowed for in navigation where errors in timekeeping of a few seconds translated into an error of longitude of miles perhaps with dire consequences. When rapid means of transport on land arrived with the railways such variation caused intolerable complications for timetables.

The solution agreed was to base the second on the year rather than the day. The exact number of days, hours, minutes and seconds in a year was known from astronomical observation. From that the exact number of seconds of which a year should consist can be precisely calculated, and so a clock could be constructed which measured time-intervals in exactly this length of second. Given a second of known length, the length of the minute, hour and day can be calculated by multiplication. The result is a clock day which does not coincide with the sundial day, because the clock does not oscillate from day to day, but which provides a regular base for timetables. Hence it is called an average or mean time.

However it was then observed using such a clock that the year itself oscillated, this time because of the elliptical nature of the Earth's orbit around the Sun amongst other things. Undoubtedly every celestial framework will give some sort of oscillation, though probably diminishing in its effect on the astronomical second, because everything in the heavens is in some sort of cyclical motion.

But modern technology may require precision of a billionth of a second or better. Hence the SI decision to define the second in terms of a completely different phenomenon, in effect the period of oscillation of the electromagnetic radiation from a particular atomic transition. This is very precise. Whether it is stable depends on the invariability of the properties of electromagnetic radiation, which may not always be a justifiable assumption everywhere.

An object of this invention is a completely new basis for timekeeping to decouple timekeeping from the second and the phenomena used to measure seconds of whatever precision. It is an object of this invention to measure time in UNIVERSAL ABSOLUTE TIME-INTERVALS which do not vary from time to time or from place to place in the Universe.

This invention comprises a methodology and apparatus for measuring UNIVERSAL ABSOLUTE TIME-INTERVALS. UNIVERSAL ABSOLUTE TIME-INTERVALS are defined as time-intervals which are independent of the variation of known natural phenomena. Thus UNIVERSAL ABSOLUTE TIME-INTERVALS represent the same measured length of elapsed time everywhere and at all times. Measurement of UNIVERSAL ABSOLUTE TIME-INTERVALS provides a common, lasting standard which can be made available to all interested parties.

The apparatus and methodology based on radioactivity of this invention provide units of time-interval that are ABSOLUTE and totally independent of all other phenomena. In principle it can be as precise as patience permits, comparable to the atomic clock or 1 in 10^{10} . The process of radioactive decay is not affected by known natural phenomena. The process of radioactive decay itself forms the basis of the measurement of time-intervals. It is an object of this invention to rebase exponential radioactive decay curves on a base of UNIVERSAL ABSOLUTE TIME-INTERVALS. It is a further object of this invention to display time-intervals directly as UNIVERSAL ABSOLUTE TIME-INTERVALS.

Radioactive decay is stochastic. The decay of any single nucleus is not a predictable event. Decay of a nucleus may occur at any time, irrespective of the base on which time is measured. Radioactive nuclei decay completely independently of each other.

However, decay in a population of large numbers of radioactive nuclei of a radioactive species is both extremely predictable and characteristic of that species. The term population is used in the statistical sense of a very large number of individuals of a species such that its behaviour encompasses that of all smaller numbers or samples of the species. In effect the behaviour of a population overrides the stochastic variations of the individuals of which the population is composed.

The number of radioactive nuclei in a population of a radioactive species which decay during an interval of time is proportional to the number of radioactive nuclei of the species present. Each radioactive nucleus of a species has the same probability that it will decay during any particular time-interval. Decay in a population of a radioactive species is therefore an exponential decline with respect to time. Any apparent variation in the rate of decay of a population of a radioactive species in excess of experimental error is caused by variations in the time-base against which it is measured.

It is an object of this invention to eliminate variation in the length of time-interval caused by variation in the time-bases used in current methods of timekeeping. It is an object of this invention to rebase measurements of decay on a base of UNIVERSAL ABSOLUTE TIME-INTERVALS.

Radioactive elements decay by a change of state of the nucleus of the individual atom from which a quantum of radiation is emitted. Each quantum of radiation is a decay event which can be individually detected and counted under suitable conditions. Radioactive nuclei which are identical are referred to as a species. Radioactive decay occurs at a rate which is a fundamental characteristic of the radioactive species. It is not known to be affected by natural phenomena in the environment, and so it is not known to be affected by their variations.

Repetition in the apparatus and methodology based on radioactive decay of this invention is the equal and constant probability of decay of individual radioactive nuclei of the same species within the apparatus. This provides a means of measuring time event by event in large populations of radioactive nuclei, expressed solely in terms of the number of decay events.

The terms radioactive atom and nucleus are conveniently used interchangeably depending on the context. Radioactive decay is a property of the nucleus. However populations of nuclei under normal conditions, as when large masses are being weighed, are always in the form of atoms because they are complete with orbital electrons.

Number having no dimensions, unlike physical phenomena, is by definition constant throughout time and hence space. Time intervals measured in terms of dimensionless numbers are therefore identical at any time in every part of the Universe. It is an object of this invention to measure UNIVERSAL ABSOLUTE PARAMETERS OF

DECAY and UNIVERSAL ABSOLUTE TIME-INTERVALS in terms of dimensionless numbers.

Measurement of time-interval by watching a single radioactive atom is a fruitless task. Starting at any particular point in time, it may decay soon, or it may decay late, depending apparently on when the atom itself decides. As far as the observer is concerned, decay occurs at random, or in terms which are more precise mathematically, the process is stochastic. All that is known is that atoms of some species are much more likely than others to decay while you are watching. So for carbon-14 half the atoms which are observed are likely to decay in 5100 years. For other species it may be much longer, and for others much shorter. There is no way of determining which atoms will constitute the half.

If the observer watches two radioactive atoms of the same species, both may decay soon, or both may decay late, but just as likely is that one atom will decay soon and one late. The time which elapses between the initial point in time and the decay of an atom, or the probability of decay of an atom, can be much more precisely estimated. It is somewhere between the two extremes.

If the observer goes on to watch a hundred atoms of the same radioactive species, it is still just possible that they will all decay soon or all decay late, but the probability is that they will be more spread out in time, and so at any particular point in time the number of decay events which have occurred will give a better chance of discerning the average time to decay which is characteristic of the species. However, a hundred atoms is not many, and each different sample of one hundred atoms will give a different number of decay events up to the same point in time, even though they must all reach the same number eventually. The exponential decay curve does not cut the time-axis. There is a compromise in practical use between precision and convenient timekeeping.

On the grounds of probability, a sample of a million atoms would give a number of decay events after a particular time-interval which was more characteristic of the decay of the radioactive species. A hundred million would be more accurate still.

Each of these numbers represents a sample from a population of atoms. A sample is defined as a number of atoms of a radioactive species which give a different count of decay events per gram from the count per gram given by a population of atoms and from each other after a definite interval of time. The population of atoms is an infinitely large number of radioactive atoms. The population subsumes the behaviour of all samples drawn from the population. A sample gives an estimate of the average time to decay with a range of precision which can be statistically calculated. The population gives the parameter which is characteristic of the radioactive species itself.

Data for the decay characteristics of radioactive species, in common with all descriptions of chemical processes, assume that the measurements have been made with populations of atoms, so that the results are universal, and not just characteristic of the samples chosen. The number of atoms in a small mass of substance is so large that this is almost always justified. This is quite different from experimental error due to technique. Even in the absence of experimental error, which is not possible to achieve in practice, samples would still produce these different outcomes by definition.

As sample mass increases so its behaviour and the estimates of average time to decay approach that of the population. By definition no sample can ever represent the population perfectly. However with increasing sample mass the estimates from samples become indistinguishable from that calculated for the population as a whole within the precision required of the answer. The precision of measurement is not sufficient at this sample mass to reveal the stochastic nature of the process of decay.

Thus if the required precision is one in a hundred, there is a sample mass at which the sample is just indistinguishable from the population, because the accuracy of the measurement is not sufficient to detect the differences. If the precision of the measurement was increased to one in a million, the differences after the same time-interval would probably become apparent.

The STOCHASTIC THRESHOLD MASS for decay of a radioactive species is defined here as the minimum mass of a sample of radioactive substance at which the behaviour of its component atoms taken as a whole becomes indistinguishable at the required precision of measurement from those of a mass which represents the population as a whole. The STOCHASTIC THRESHOLD MASS for decay of a radioactive species depends on numbers of radioactive nuclei per unit mass and so its atomic weight.

In a process which depends on chance, it is always theoretically possible, however unlikely, that a single sample could show behaviour identical to that of a population. However, the chances of this occurring in a number of samples is negligibly small, and so replication with a number of samples eliminates this possibility entirely.

To measure UNIVERSAL ABSOLUTE TIME-INTERVALS by radioactivity a radioactive isotope is selected which decays slowly enough to count the decay events with the required accuracy, but fast enough to reach a result in a time acceptable for the application. The isotope selected gives off radiation which is unambiguous to detect and easy to count and so probably a single species. It depends on the ability of detector counters to discriminate between separate events.

The apparatus of this invention uses as large a mass as possible of a radioactive substance which is well characterised both qualitatively and quantitatively. The composition of the substance is measured by mass spectroscopy or other analytical methods to determine the exact proportion of radioactive atoms which it contains. The quantity of this substance which is placed in the apparatus is measured as precisely as possible by exact methods of weighing or other methods. The mass of radioactive atoms which this mass of substance contains is calculated from the mass of the substance and the proportion of radioactive species. Not all the atoms of the mass of radioactive substance may be radioactive. The number of radioactive atoms is calculated from the mass of the radioactive atoms and their atomic weight.

The radioactive substance may be in gas, liquid or solid form. The radioactive species is chosen for its suitable decay rate and its availability in consistent form. Suitable substances are cobalt-60, strontium-90, americium-241 and carbon-14. Their decay gives different types of emission, such as alpha-, beta- and gamma-radiation. Each particle or ray emitted is the result of decay of a radioactive nucleus. Decay of a radioactive nucleus is a decay event. Decay events may be detected and counted.

The apparatus of this invention for measuring UNIVERSAL ABSOLUTE TIME-INTERVALS by radioactivity is designed to capture and count every particle or ray emitted from the radioactive mass for the lifetime of the apparatus. Suitable apparatus is radioactivity measurement equipment used for low level counting incorporating special modifications designed to yield greater detection efficiencies and lower background counting rates than those necessary for normal applications. Such apparatus usually totally encloses the substance and the detector counter to trap and count radiation which is emitted at all angles.

The apparatus is shielded to exclude as much extraneous radiation as possible from other sources to improve the quality of the count. What background radiation still gets through may be counted by an anti-coincidence system and electronically subtracted from the count to obtain the count of decay events due to the mass of radioactive substance itself.

Another method may be to maintain at a constant level the flux of relevant radiation from outside the radioactive mass used in the clock.

Any type of detector counter may be used to count decay events. The type of detector counter chosen depends on the type of radiation to be detected which itself depends on the radioactive substance chosen. Particularly suitable are scintillation detector counters because of the extremely high rates of emission which they are capable of detecting and counting accurately, for example $>10^5$ s^{-1}.

The procedure is to count the number of decay events in the radioactive mass one by one up to a cumulative number which depends on the required precision of the clock. This might be 10^5 for some applications, but it might be as high as 10^{10} or more. The precision of the count increases as the cumulative count increases up to point at which the STOCHASTIC THRESHOLD MASS for radioactive decay of the depleted substance is reached.

The assumption is that a large radioactive mass such as that used so far is well above this STOCHASTIC THRESHOLD MASS. If this is in doubt, it can be checked by replication against another equally large radioactive mass of the same substance.

Different masses of the same radioactive species start at different counts, whether or not their decay characteristics are the same, because a larger number of radioactive nuclei produces a proportionately larger number of decay events. For the purposes of determining the STOCHASTIC THRESHOLD MASS, which requires discrimination between population and sample behaviour, the counts of different sizes are normalised by dividing by the accurately determined starting masses of the samples. This reduces them to counts per gram of starting radioactive substance.

The next step is to take at least six aliquots of approximately equal mass of the same radioactive substance, say about a tenth of the initial mass, and to repeat the same procedure until the average count per gram reaches the count per gram obtained with the initial large mass which represents the population.

If the aliquot mass is above the STOCHASTIC THRESHOLD MASS, the value of half the standard deviation of the six normalised counts is small, such that it is within the desired limit of accuracy. For example it may be that the value is 8.7 counts in 10^6 where precision of 1 in 10^5 is required. For greater certainty, say 95% confidence a whole standard deviation is used, or for 99% confidence one and a half standard deviations. The width of the distribution is halved because the standard deviation curve is two sided.

However, if the aliquot mass is too small, the range of normalised counts is found to be larger and the value of half the standard deviation is unacceptably large for the required precision. The aliquot mass must then be increased and the procedure repeated until an acceptable value of half the standard deviation of the normalised counts for the six aliquots is obtained.

If the aliquot mass is above the STOCHASTIC THRESHOLD MASS, there is no way of judging by how much. The aliquot mass is therefore reduced, say by a factor of ten, and the procedure repeated. If the value of half the standard deviation of normalised counts is now too large, further measurements are made with aliquots having masses which are between the two aliquot masses, and so on, until the threshold mass at which the increase began is found. This is the STOCHASTIC THRESHOLD MASS.

However, if the aliquot mass is still too large after reducing it by a factor of ten, the mass is reduced by a further factor of ten, and the procedure repeated until the STOCHASTIC THRESHOLD MASS is found.

When the STOCHASTIC THRESHOLD MASS has been identified, the mass chosen to start the count of UNIVERSAL ABSOLUTE TIME-INTERVALS is much larger than this threshold to ensure that the clock does not fall below the STOCHASTIC THRESHOLD MASS of the depleted substance during the planned lifetime of the apparatus. The starting mass is measured and the substance analysed as accurately as possible, so that the initial number of radioactive atoms r_0 can be calculated.

When the count of decay events reaches a suitable total determined by the precision required, this is chosen as the count which marks the end of the first UNIVERSAL ABSOLUTE TIME-INTERVAL. This UNIVERSAL ABSOLUTE TIME-INTERVAL is designated by the number N_1.

The general equation for radioactive decay is an exponential of the form:

$$y = ke^{-\lambda t}$$

The equation relates to the decrease in the number of radioactive nuclei over time. Conventionally all decay constants λ of radioactive species are measured and quoted in the SI system of time-intervals. The ‘constants’ are therefore susceptible to variations in the duration of the second. In effect they are specific to the time and place at which they were measured.

It is a feature of this invention that the methodology and apparatus measure DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETERS OF DECAY of radioactive species as follows.

If the number of radioactive atoms present initially is r_0, then the number of radioactive atoms r_t present at any time t during the decay is given by the equation:

$$r_t = r_o e^{-\lambda t}$$

Thus after 1 unit of time-interval t=1 measured in any units from the start of the process, the number of radioactive atoms decreases from r_0 to r_1 where

$$r_1 = r_o e^{-\lambda}$$

The DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETERS OF DECAY λ is calculated as follows.

By rearrangement of the above equation

$$r_1/r_o = e^{-\lambda}$$

and

$$-\lambda = \ln(r_1/r_o)$$

The number of radioactive atoms which has decayed during the first unit of time-interval in this apparatus is $r_0 - r_1$. This is identical to the number of decay events during this period, which by the process described above for the single UNIVERSAL ABSOLUTE TIME-INTERVAL we have designated N_1. Thus

$$r_0 - r_1 = N_1$$

and

$$r_1 = r_0 - N_1$$

Then by substitution,

$$-\lambda = \ln((r_0 - N_1)/r_o)$$

or

$$-\lambda = \ln(1 - N_1/r_0)$$

From this equation λ can be calculated since N_1 has been chosen and r_0 is known from the initial assay.

The parameter λ has no dimensions. It is simply a number characteristic of the radioactive species which is decaying. Here it is termed the DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETER OF DECAY of the particular radioactive species.

From the equation, the initial number of radioactive atoms r_0 and the parameter λ, the counts of decay events at which the subsequent UNIVERSAL ABSOLUTE TIME-INTERVALS as shown in the Figure will occur can be calculated as follows.

During the first UNIVERSAL ABSOLUTE TIME-INTERVAL, the number of radioactive atoms which has decayed is $r_0 - r_1$, which we have designated N_1. Therefore

$$N_1 = r_0 - r_0e^{-\lambda}$$

or

$$N_1 = r_0(1 - e^{-\lambda})$$

During the second UNIVERSAL ABSOLUTE TIME-INTERVAL, which is consecutive with the first, the number of radioactive atoms which decay is $r_1 - r_2$ where

$$r_1 = r_0e^{-\lambda}$$

and

$$r_2 = r_0e^{-2\lambda}$$

Hence

$$\begin{aligned} r_1 - r_2 &= r_0e^{-\lambda} - r_0e^{-2\lambda} \\ &= r_0(e^{-\lambda} - e^{-2\lambda}) \\ &= r_0\, e^{-\lambda}(1 - e^{-\lambda}) \end{aligned}$$

But from the equation above

$$r_0(1 - e^{-\lambda}) = N_1$$

Therefore

$$r_1 - r_2 = N_1e^{-\lambda}$$

The number of radioactive atoms which decayed during the consecutive second UNIVERSAL ABSOLUTE TIME-INTERVAL is therefore $N_1e^{-\lambda}$. Since this is identical to the number of decay events which occurred during the period, the length of the second UNIVERSAL ABSOLUTE TIME-INTERVAL in this series can be measured by counting this number of decay events.

During the third UNIVERSAL ABSOLUTE TIME-INTERVAL, which is consecutive with the second, the number of radioactive atoms which decay is $r_2 - r_3$ where

$$r_2 = r_0e^{-2\lambda}$$

and

$$r_3 = r_0e^{-3\lambda}$$

Hence

$$r_2 - r_3 = r_0e^{-2\lambda} - r_0e^{-3\lambda}$$

$$= r_0(e^{-2\lambda} - e^{-3\lambda})$$

$$= r_0\, e^{-2\lambda}(1 - e^{-\lambda})$$

But from the equation above

$$r_0(1 - e^{-\lambda}) = N_1$$

Therefore

$$r_2 - r_3 = N_1e^{-2\lambda}$$

The number of radioactive atoms which decayed during the consecutive third UNIVERSAL ABSOLUTE TIME-INTERVAL is therefore $N_1e^{-2\lambda}$. Since this is identical to the number of decay events which occurred during the period, the length of the third UNIVERSAL ABSOLUTE TIME-INTERVAL in this series can be measured by measuring this number of decay events.

To generalise, during the nth UNIVERSAL ABSOLUTE TIME-INTERVAL, which is consecutive with the (n-1)th UNIVERSAL ABSOLUTE TIME-INTERVAL, the number of radioactive atoms which decay is $r_{(n-1)} - r_n$ where

$$r_{(n-1)} = r_0e^{-(n-1)\lambda}$$

and

$$r_n = r_0e^{-n\lambda}$$

Hence

$$r_{(n-1)} - r_n = r_0e^{-(n-1)\lambda} - r_0e^{-n\lambda}$$

$$= r_0(e^{-(n-1)\lambda} - e^{-n\lambda})$$

$$= r_0\, e^{-(n-1)\lambda}(1 - e^{-\lambda})$$

But from the equation above

$$r_0(1 - e^{-\lambda}) = N_1$$

Therefore

$$r_{(n-1)} - r_n = N_1e^{-(n-1)\lambda}$$

The number of radioactive atoms which decayed during the nth consecutive UNIVERSAL ABSOLUTE TIME-INTERVAL is therefore $N_1e^{-(n-1)\lambda}$. As before, the length of the nth consecutive UNIVERSAL ABSOLUTE TIME-INTERVAL in this series can be measured by measuring this number of decay events.

UNIVERSAL ABSOLUTE TIME-INTERVAL	Number of Decay Events during the Period	Cumulative Number of Decay Events at End of TIME-INTERVAL
1	N_1	N_1
2	$N_1e^{-\lambda}$	$N_1(1 + e^{-\lambda})$
3	$N_1e^{-2\lambda}$	$N_1(1 + e^{-\lambda} + e^{-2\lambda})$
…	…	…
…	…	…
n	$N_1e^{-(n-1)\lambda}$	$N_1(1 + e^{-\lambda} + e^{-2\lambda} + \ldots\ldots + e^{-(n-1)\lambda})$

Table. Increase in Number of Decay Events with Consecutive

UNIVERSAL ABSOLUTE TIME-INTERVALS

The cumulative total of decay events during the decay process is the sum of the decay events measured in the individual time-intervals. Thus:

- the 1st UNIVERSAL ABSOLUTE TIME-INTERVAL ends when the count of decay events which have occurred since the start of the clock reaches N_1 (definition).
- the 2nd UNIVERSAL ABSOLUTE TIME-INTERVAL ends when the count of decay events which have occurred since the start of the second unit reaches $N_1e^{-\lambda}$, when the cumulative total is $N_1 + N_1e^{-\lambda}$ or

$$N_1(1+ e^{-\lambda})$$

- the 3rd UNIVERSAL ABSOLUTE TIME-INTERVAL ends when the count of decay events which have occurred since the start of the third the unit reaches $N_1e^{-2\lambda}$, when the cumulative total is $N_1 + N_1e^{-\lambda} + N_1e^{-2\lambda}$ or

$$N_1(1+ e^{-\lambda} + e^{-2\lambda})$$

- the nth UNIVERSAL ABSOLUTE TIME-INTERVAL ends when the count of decay events which has occurred since the start of the nth unit reaches $N_1e^{-(n-1)\lambda}$ when by analogy the cumulative total is

$$N_1(1+ e^{-\lambda} + e^{-2\lambda} + \ldots\ldots\ldots + e^{-(n-1)\lambda})$$

These results are summarised in the Table.

Thus from N_1 and λ the counts of decay events which will mark the end of UNIVERSAL ABSOLUTE TIME-INTERVAL can be predicted as far as the point at which the STOCHASTIC THRESHOLD MASS of the product of depletion by decay is reached. This may be comparable to the half-lives of common radioactive substances which are measured in terms of thousands of years. The succession of consecutive UNIVERSAL ABSOLUTE TIME-INTERVALS is predictable in practical terms for the indefinite future.

There are several different modes of use of radioactive timekeeping of this invention. The argument has been developed on the basis of starting with a known number of radioactive atoms and following their decay. The accuracy of timekeeping then depends on the accuracy of the initial estimate of this number. This requires quantitative analysis of the starting substance as accurately as possible with currently available techniques, almost certainly using mass spectrometry, and accurate weighing or other technique of the substance characterised in this way, in order to measure the starting mass. Knowledge of the mass, composition and atomic weight allows the number of atoms to be calculated with an error which contains all the errors from these different sources of variation.

If a second apparatus were made with the same radioactive substance to match the first apparatus as closely as possible, there would be a difference between the time-intervals shown by the two sets of apparatus which depended on the different experimental errors of the initial analyses in the two cases. This might limit the accuracy of sharing UNIVERSAL ABSOLUTE TIME-INTERVALS even among apparatus of the same design.

An alternative is to agree a single apparatus as the UNIVERSAL STANDARD RADIOACTIVE CLOCK which would be as well characterised as possible, and which would provide the STANDARD UNIVERSAL ABSOLUTE TIME-INTERVAL for a very long time. This is a parallel with the kilogramme unit of mass.

Such a STANDARD UNIVERSAL ABSOLUTE TIME-INTERVAL can be used to calibrate other apparatus made in the same way with the same radioactive species. This applies even if they vary in design, and use different, probably smaller quantities of radioactive substance, while remaining above the STOCHASTIC THRESHOLD

MASS for decay of that particular radioactive species and its depleted product for their design lives. Such apparatus is referred to here for clarity as a daughter clock.

Daughter clocks fall into two classes: one which uses the same radioactive species, and one which uses a completely different radioactive species.

If the radioactive species is the same as in the UNIVERSAL STANDARD RADIOACTIVE CLOCK, it will have the same DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETER OF DECAY λ, which is characteristic of the species, but it will have a different starting number of radioactive atoms. It will in effect start lower down the decay curve.

This initial mass of radioactive atoms may be measured by assay, as with the UNIVERSAL STANDARD RADIOACTIVE CLOCK itself, which incurs experimental errors.

It may be more precisely measured by running the apparatus in parallel with the UNIVERSAL STANDARD RADIOACTIVE CLOCK itself. Let one UNIVERSAL ABSOLUTE TIME-INTERVAL on the UNIVERSAL STANDARD RADIOACTIVE CLOCK correspond to a count of $N_{d,1}$ on the daughter clock, where d is for daughter. Then counts on the daughter clock corresponding to UNIVERSAL ABSOLUTE TIME-INTERVALS on the UNIVERSAL STANDARD RADIOACTIVE CLOCK are calculated from the Table by using the same value for the DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETER OF DECAY λ as for the UNIVERSAL STANDARD RADIOACTIVE CLOCK, but substituting $N_{d,1}$ for N_1. This procedure avoids incurring experimental errors in the characterisation of the radioactive substance in the daughter clock. In effect the procedure simply transfers UNIVERSAL ABSOLUTE TIME-INTERVALS from the UNIVERSAL STANDARD RADIOACTIVE CLOCK to the daughter at the required precision without increasing experimental error.

If the radioactive species is different from that used in the UNIVERSAL STANDARD RADIOACTIVE CLOCK, it is necessary to characterise its decay curve completely by calculating both its DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETER OF DECAY λ_d and the count $N_{d,1}$ corresponding the Standard UNIVERSAL ABSOLUTE TIME-INTERVAL on the UNIVERSAL STANDARD RADIOACTIVE CLOCK.

This can be done by assay as before, which incurs experimental errors.

The UNIVERSAL ABSOLUTE TIME-INTERVAL may be transferred from the UNIVERSAL STANDARD RADIOACTIVE CLOCK to the daughter by running them in parallel for three complete consecutive UNIVERSAL ABSOLUTE TIME-INTERVALS as measured on the UNIVERSAL STANDARD RADIOACTIVE CLOCK, and noting the counts on the daughter clock at which these occur.

Let the daughter clock begin with a number of radioactive atoms $r_{d,0}$ which is an unknown.

Let the number of radioactive atoms in the daughter clock decrease as in the Figure to $r_{d,1}$ at the end of the first UNIVERSAL ABSOLUTE TIME-INTERVAL, to $r_{d,2}$ at the end of the second UNIVERSAL ABSOLUTE TIME-INTERVAL and to $r_{d,3}$ at the end of the third UNIVERSAL ABSOLUTE TIME-INTERVAL, where UNIVERSAL ABSOLUTE TIME-INTERVALS are shown on the UNIVERSAL STANDARD RADIOACTIVE CLOCK. By definition

$$r_{d,0} - r_{d,1} = N_{d,1}$$

which is the first parameter we need to know. Then from the Table using alternative expressions for exponential functions

$$r_{d,1} - r_{d,2} = N_{d,1} \exp(-\lambda_d)$$

and

$$r_{d,2} - r_{d,3} = N_{d,1} \exp(-2\lambda_d)$$

from which

$$(r_{d,1} - r_{d,2})/(r_{d,2} - r_{d,3}) = N_{d,1} \exp(-\lambda_d)/N_{d,1} \exp(-2\lambda_d)$$

which by cancellation gives

$$(r_{d,1} - r_{d,2})/ r_{d,2} - r_{d,3}) = \exp(-\lambda_d)/\exp(-2\lambda_d)$$

$$= \exp(\lambda_d)$$

From this

$$\lambda_d = \ln((r_{d,1} - r_{d,2})/ (r_{d,2} - r_{d,3}))$$

But the right hand side of the equation is the ratio of the number of counts of decay events which occurred on the daughter clock during the second and third UNIVERSAL ABSOLUTE TIME-INTERVALS. So the DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETER OF DECAY for the daughter clock λ_d is the natural logarithm of the ratio of daughter clock counts during the second and third STANDARD UNIVERSAL ABSOLUTE TIME-INTERVALS as measured on the UNIVERSAL STANDARD RADIOACTIVE CLOCK.

Counts on the daughter clock which correspond to UNIVERSAL ABSOLUTE TIME-INTERVALS measured on the UNIVERSAL STANDARD RADIOACTIVE CLOCK are then calculated from the Table substituting $N_{d,1}$ for N_1 and λ_d for λ.

Radioactive species different from the standard are chosen to match other applications which require a longer or shorter lived clock, though they must also give decay events which can be unambiguously counted. The only practical consideration is that their useful time bases must overlap sufficiently to allow the above procedures to be carried out.

However some radioactive species might have largely decayed before the UNIVERSAL STANDARD RADIOACTIVE CLOCK reaches the end of the STANDARD UNIVERSAL ABSOLUTE TIME-INTERVALS. In this case the UNIVERSAL ABSOLUTE TIME-INTERVAL can be transferred by means of intermediate clocks, using other radioactive species, which bridge the gap. Since nothing is involved other than numbers of counts, no errors are introduced in the transfer from the UNIVERSAL STANDARD RADIOACTIVE CLOCK.

It is a feature of the radioactive clocks of this invention that the value of the DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETER OF DECAY can be calculated with ever increasing precision as the clock runs, because a new estimate can be calculated from the counts which mark the beginning and the end of each time-interval passed. Each estimate can be combined with previous estimates to give a running average which ever more closely approximates to the ultimate value for the clock.

Alternatively, the UNIVERSAL ABSOLUTE TIME-INTERVAL may itself be redefined to a more convenient length as the count increases in order to take account of the increasing precision of the count. For example the UNIVERSAL ABSOLUTE TIME-INTERVAL may be increased to cover , say, 10 UNIVERSAL ABSOLUTE TIME-INTERVALS. It is a question of recalculation of the accumulating data.

Translation from counts to UNIVERSAL ABSOLUTE TIME-INTERVAL is an exponential function which is readily computed after calculation of the DIMENSIONLESS UNIVERSAL ABSOLUTE PARAMETER OF DECAY for the individual clock. The result can be displayed on the clock as UNIVERSAL ABSOLUTE TIME-INTERVALS, individual or cumulative, without showing the counts at all.

Subdivisions of the UNIVERSAL ABSOLUTE TIME-INTERVAL can be calculated but they may lose precision compared with the first UNIVERSAL ABSOLUTE TIME-INTERVAL because the number of decay events is smaller.

It is also possible to run clocks with radioactive species of very different half lives, such that the faster decaying radioactive species can provide convenient subdivisions of the UNIVERSAL ABSOLUTE TIME-INTERVALS of the UNIVERSAL STANDARD RADIOACTIVE CLOCK. These may have smaller atomic weights and so more decay events for each UNIVERSAL ABSOLUTE TIME-INTERVAL. It depends on the ability of sensors to count the greater number of decay events with the required precision.

A daughter clock can be started and calibrated at any time during the life of the UNIVERSAL STANDARD RADIOACTIVE CLOCK, so that the unit of UNIVERSAL ABSOLUTE TIME-INTERVAL can be passed from clock to clock over very long periods of time without loss of accuracy.

There may be an advantage in making the chosen UNIVERSAL STANDARD RADIOACTIVE CLOCK obviously different from the SI unit of time-interval which is the second. There is no reason to try to approximate to the second, as has been done with other clocks.

It is possible to run conventional clocks, such as the caesium-133 clock, in parallel with the UNIVERSAL STANDARD RADIOACTIVE CLOCK to compare time bases. If the caesium-133 clock recorded a time-interval of x seconds during n successive UNIVERSAL ABSOLUTE TIME-INTERVALS on the UNIVERSAL STANDARD RADIOACTIVE CLOCK, then

n UNIVERSAL ABSOLUTE TIME-INTERVALS = x caesium-133 clock seconds

or

1 UNIVERSAL ABSOLUTE TIME-INTERVAL = x/n caesium-133 clock seconds

at that particular time and place. If that ratio varied, it must be because the caesium-133 clock second had varied under the influence of the environment in which the comparison was made.

The UNIVERSAL STANDARD RADIOACTIVE CLOCK may be constructed with a very long working life comparable to the half life of some common radioactive species, which are measured in thousands of years. This may be prolonged indefinitely by the process of transferring UNIVERSAL ABSOLUTE TIME-INTERVALS described above.

Scintillator substances, detector counters and measuring techniques are improving all the time, which may allow greater sensitivity and precision, and improve and extend the operation of clocks of this invention.

In the methodology and apparatus for measuring UNIVERSAL ABSOLUTE TIME-INTERVALS by the decay of radioactive substances of this invention the probability of decay of each radioactive nucleus of a radioactive species is the same and constant.